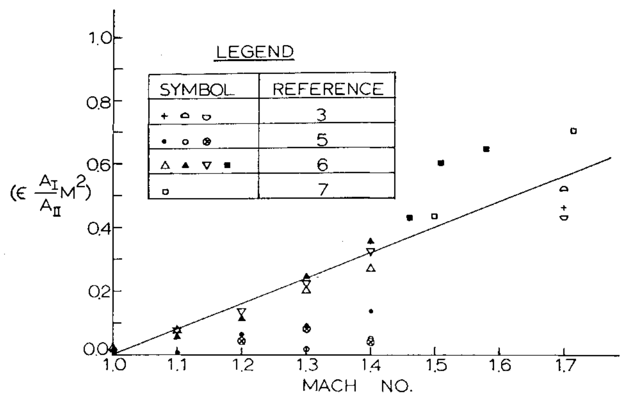

Fig. 2 Straight line approximation in the correction factor determination.

The resulting value for f is then

$$f = f_n + 2\,\frac{\left[1 + \left(\frac{\gamma - 1}{2}\right)\eta_0 f_n M_s^2\right]}{\gamma \eta_0 M_s^2} \times \left[\frac{g(M_1,\eta_0,f)}{g(M_1,\eta_0,f_n)} - 1\right] \quad (10)$$

The results of Eq. (9) for various experimental data are plotted vs initial Mach number in Fig. 2. The plot of the product $\epsilon(A_{\mathrm{I}}/A_{\mathrm{II}})M_1^2$ vs Mach number is presented since it can be readily approximated by a straight line. The data used here represents a wide range of cascade geometries and test conditions as described in Table 1. The error represented by the scatter in the data about the straight line represents no more than $\pm 10\%$ in the value of the total pressure ratio $(P_t)_2/(P_t)_1$. The value of the correction factor based on the straight line fit of the data with f_n equal unity is

$$f = 1 + \tfrac{4}{5}(A_{\mathrm{II}}/A_{\mathrm{I}})[(1/M_1^2) - 1/M_1] \quad (11)$$

Substitution of the results of Eqs. (3) and (11) for η_0 and f, respectively, into Eq. (5) results in the desired semiempirical relationship from which the total pressure ratio $(P_t)_2/(P_t)_1$ can be estimated.

IV. Concluding Remarks

A semiempirical expression for the prediction of supersonic compressor efficiency has been developed. The result is based on an assumed shock model and total pressure data taken from experimental tests on cascades of varying geometrical properties. The fact that the data can be reasonably represented by a straight line as a function of inlet Mach number makes the resulting efficiency expression attractive for design purposes.

References

[1] Der, J. et al., "Numerical Analysis of Supersonic Flow Through Curved Channels," ARL 63-117, 1963, Aerospace Research Laboratories, Office of Aerospace Research, Wright-Patterson Air Force Base, Ohio.

[2] Der, J. and Mullings, B. N., "Numerical Methods and Fortran Program for Flow Field Calculations in Supersonic Cascades," ARL 64-164, 1964, Aerospace Research Laboratories, Office of Aerospace Research, Wright-Patterson Air Force Base, Ohio.

[3] Hangen, R. L. and Steurer, J. W., "Cascade Testing of Supersonic Compressor Blade Elements," ARL 69-0034, 1969, Aerospace Research Laboratories, Office of Aerospace Research, Wright-Patterson Air Force Base, Ohio.

[4] Cresci, R. J., "An Investigation of a Hypersonic Cascade with 120° Turning Passages," ARL 63-168, 1963, Aerospace Research Laboratories, Office of Aerospace Research, Wright-Patterson Air Force Base, Ohio.

[5] Starken, H. and Lichtfuss, H. J., "Some Experimental Results of Two-Dimensional Compressor Cascades at Supersonic Inlet Velocities," ASME Paper 70-GT-7, Gas Turbine Conference and Products Show, Brussels, Belgium, May 24–28, 1970.

[6] Heilmann, W., Starken, H., and Weyer, H., "Cascade Wind Tunnel Tests on Blades Designed for Transonic and Supersonic Compressors," Paper 11, AGARD Conference Proceedings No. 34, Sept. 1968.

[7] Hermann, P., "Further Investigations of a Blunt Trailing Edge Cascade in the S-3 Supersonic Wind Tunnel," Internal Note 9, June 1964, von Kármán Institute for Fluid Dynamics, Rhode-Saint-Genese, Belgium.

[8] Balzer, R. L., "A Method for Predicting Compressor Cascade Total Pressure Losses When the Inlet Relative Mach Number is Greater than Unity," ASME Paper 70-GT-57, Gas Turbine Conference and Products Show, Brussels, Belgium, May 24–28, 1970.

[9] Boxer, E., "A Method for Predicting the Performance of High Reaction Supersonic Compressor Blade Sections," AIAA Paper 69-522, Colorado Springs, Colo., 1969.

[10] Losey, D. and Tabakoff, W., "A Technique for Estimating Axial Flow Compressor Potential Peak Efficiency and Related Performance," *Journal of Aircraft*, Vol. 4, No. 2, March–April, 1967, pp. 133–136.

Optimal Stochastic Control and Aircraft Gust Alleviation

Ronald A. Hess*
U.S. Naval Postgraduate School, Monterey, Calif.

Nomenclature

a_{c_z}	= linear acceleration of aircraft center of mass in z direction, ft/sec^2
C_{L_0}	= lift coefficient at trim angle of attack
g	= local acceleration due to gravity, ft/sec^2
h	= aircraft altitude, ft
$K_1 \rightarrow K_7$	= control system gains
L_w	= gust characteristic length, ft
n	= $-a_{c_z}/g$, normal acceleration factor
$\dot{q}$	= aircraft pitch rate
U_0	= aircraft forward velocity, fps
w_g	= gust vertical velocity, fps
xyz	= aircraft stability axis system (Ref. 2)
α_g	= $-w_g/U_0$, angle of attack perturbation due to gust, measured at aircraft center of mass, rad
η	= elevator angle, measured from trim, rad
Ω	= spatial frequency, rad/ft
σ_w	= rms vertical gust velocity, fps

Introduction

THE description of random atmospheric turbulence using semiempirical spectral models has facilitated the problem of calculating aircraft responses for flight through turbulent air. The design of automatic flight control systems to minimize mean square aircraft accelerations and to suppress certain elastic structural modes in level flight has been investigated. Little work has been done, however, in relating the optimal control system gains to the spectral characteristics of the gust field.

The research summarized in this Note has been directed toward the problem of minimizing the mean square normal acceleration of the center of mass of a large, rigid, jet transport flying level through a one-dimensional, turbulent upwash field.[1] In particular, the dependence of the mean square performance and optimal control system gains upon the gust

Received November 3, 1970.

* Assistant Professor, Department of Aeronautics. Associate Member AIAA.

characteristic length has been noted. The strength of this dependence determines the utility of "on-line" measurement of the gust characteristic length for the purposes of implementing an adaptive control policy.

Analysis

The mathematical models of aircraft and turbulence have been kept as simple as possible. The aircraft is identical to the large, rigid, jet transport utilized in Etkin.[2] The assumption of rigidity is not as restrictive as it may seem. It has been shown that the suppression of rigid body modes is of prime importance from both a handling qualities and a structural standpoint.[3]

Two flight conditions were studied: a 30,000 ft, 500 mph cruise condition and a sea level, 200 mph landing approach. The one-dimensional upwash field excites only the longitudinal modes and the contribution of the phugoid has been neglected. Only first-order gust effects have been considered, i.e., the gust perturbation is considered to be "aerodynamically equivalent" to an angle of attack perturbation α_g, and a pitch rate perturbation $\hat{q}_g$. The elevator was the only control mechanism utilized.

A stationary, homogeneous, isotropic, frozen turbulence field was assumed and a simplified power spectrum used:

$$\Phi_{w_g}(\Omega) = 2\sigma_w{}^2 L_w \, 1/[1 + (L_w\Omega)^2]$$

Although this form has no theoretical basis, its agreement with measured spectra, the preservation of the characteristic length L_w and its simplicity suggest its use.[4] The rms gust intensity σ_w was chosen as 10 fps in all cases. The index of performance was given by

$$J = \lim_{T\to\infty} \frac{1}{T} \int_0^T [qn^2(t) + ru^2(t)]dt = \overline{qn^2} + \overline{ru^2}$$

where $u(t)$ is the signal driving the elevator-servo combination, here approximated by a first-order lag with a time constant on the order of 0.1 sec. For the range of gust intensities normally encountered ($\sigma_w = 0 \rightarrow 40$ fps) and for $q/r \leq 10$, the gust intensity has no effect upon the form of the optimal regulator.

Figure 1 shows the optimal flight control system. Gains K_1, K_2, and K_3 are functions of the flight condition only, i.e., altitude h, velocity U_0, and trim angle lift coefficient C_{L_0}. Gain K_4, however, depends upon the gust Characteristic length L_w, in addition to the flight condition.

The root square locus technique was utilized to select q/r, the ratio of the index of performance weighting factors. The root square locus technique, which displays the optimal closed loop roots as a function of q/r, allows the designer to trade off control deflection with transient response.[5] Once a value of q/r has been selected, the optimum values of $K_1 \rightarrow K_3$ are uniquely determined. In this study the optimum gain values were found via the steady-state matrix Riccati equation.

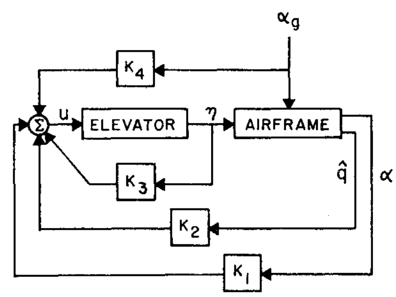

Fig. 1 Block diagram for optimal gust alleviation system.

In order to determine the utility of a gust adaptive approach, the performance of the optimal system was compared to that of two suboptimal systems. The first considered $K_4 = 0$, i.e., no gust measurement, and the second considered K_4 as an average of the optimum values over the range of L_w's (500–6000 ft) for each flight condition. In both of these cases, gains $K_1 \rightarrow K_3$ take on their optimum values as determined by the flight condition. Considering $K_4 = 0$ is analytically equivalent to assuming a white noise gust spectrum. Using an average K_4 for a given flight condition obviates adaption to changes in, or uncertainty about L_w. In both of these suboptimal approaches, the entire set of gains are functions of the flight condition alone.

Table 1 summarizes the performance data. For $K_4 = 0$, the mean square performance was worse than the unalleviated or open loop case for all $L_w > 1000$ ft. The second suboptimal approach was much more satisfactory and resulted in an average 46% reduction in mean square normal acceleration as compared to the open loop. The quality of this latter approach is evident when compared to the optimum average reduction of 49%.

Table 1 Comparison of optimal and suboptimal mean square normal acceleration with open loop mean square normal acceleration

$q/r = 10$. $K_1 = K_{1_{opt}}$	$K_2 = K_{2_{opt}}$	$K_3 = K_{3_{opt}}$				
	Performance increment in cruise			Performance increment in approach		
L_w ft	$K_4 = K_{4_{opt}}$	$K_4 = 0$	$K_4 = K_{4_{avg}}$	$K_4 = K_{4_{opt}}$	$K_4 = 0$	$K_4 = K_{4_{avg}}$
500	42%	25%	37%	38%	18%	35%
1000	49%	0%	48%	43%	...	43%
2000	53%	...[a]	53%	46%	...	46%
3000	54%	...	53%	47%	...	46%
4000	54%	...	52%	47%	...	46%
5000	54%	...	51%	48%	...	45%
6000	55%	...	50%	48%	...	44%

[a] ... indicates larger mean square normal acceleration than the open loop case.

Indirect Gust Measurement

The analysis, as summarized, assumed exact, instantaneous knowledge of the state variables, including α_g. On-line determination of α_g using aerodynamic devices, such as incidence vanes and pressure probes, does not appear feasible. It is this inability to find a suitable gust sensor which has forced recent gust alleviation studies[6] to abandon gust feed-forward signals such as proposed here.

A possible alternative to direct measurement of α_g involves utilizing aircraft motion to indirectly determine the gust perturbation. Since the derivatives $C_{z_{\dot{\alpha}}}$ and C_{z_q} were negligible in comparison to C_{z_α} and C_{z_η}, it was found that the gust angle of attack perturbation could be given by

$$\alpha_g = K_5 n + K_6 \alpha + K_7 \eta$$

where $K_5 \rightarrow K_7$ are functions of the flight condition. The variables n, α, and η can be measured much more accurately than can α_g itself. Thus, in theory at least, the feed-forward signal can be replaced by the three feedback signals previously mentioned.

Conclusions

The simplified feasibility study previously outlined has produced the following results:

1) Indirect, "on-line" gust measurement is a feasible method for improving the performance of an automatic flight control system.

2) The performance increment obtained from on-line knowledge of the spectral properties of the gust field (characteristic length, L_w) does not justify the additional system complexity necessary to obtain such information. This means that the feedback gains can be considered as functions of the flight condition alone.

3) The performance increment obtained from on-line knowledge of the flight condition does justify an adaptive

approach. A gain scheduling program would be a simple means of accomplishing this adaption.

4) As compared to the unalleviated or open loop configuration, utilizing optimal stochastic control theory and the root square locus concept resulted in an average 46% reduction in the mean square normal acceleration factor and an average 27% increase in the damping ratio of the short period roots.

References

[1] Hess, R. A., "An Application of Optimal Stochastic Control Theory to the Problem of Aircraft Gust Alleviation," Ph.D. dissertation, Aug. 1970, University of Cincinnati, Cincinnati, Ohio.

[2] Etkin, B., *Dynamics of Flight*, Wiley, New York, 1963.

[3] Austin, W. H. Jr. and Griffin, J. M., "The Interaction of Handling Qualities, Stability, Control, and Structural Loads," AGARD Advisory Report 16, Nov. 1968.

[4] Hart, J. E., Adkins, L. A., and Lacua, L. L., "Stochastic Disturbance Data for Flight Control System Analysis," ASD-TDR-62-347, Sept. 1962, Air Force Flight Dynamics Lab., Wright-Patterson Air Force Base, Ohio, p. 67.

[5] Rynaski, E. G. and Whitbeck, R. F., "The Theory and Application of Linear Optimal Control," AFFDL-TR-65-28, Jan. 1966, Air Force Flight Dynamics Lab., Wright-Patterson Air Force Base, Ohio.

[6] "Aircraft Load Alleviation and Mode Stabilizatiou (LAMS)," AFFDL-TR-68-158, Dec. 1968, Air Force Flight Dynamics Lab., Wright-Patterson Air Force Base, Ohio.

Center Deflections of Square Plates with Elastic Edge Beams

JAMES TING-SHUN WANG*

Georgia Institute of Technology, Atlanta, Ga.

Nomenclature

a	= half plate width
D	= flexural rigidity of a plate
E	= modulus of elasticity of plate material
E_b	= modulus of elasticity of edge beam material
EH	= extensional rigidity of a plate
h	= total plate thickness
I	= moment of inertia of the edge cross-sectional area
N	= load on edge beams
p	= load on plate
t	= face sheet thickness
U	= edge beam displacement
U_o	= edge beam midsection displacement
$\bar{u},\bar{v},\bar{w}$	= displacement components of a membrane with rigid edge beams
W_L	= largest transverse displacement
W_s	= smallest transverse displacement
W_o	= transverse displacement of the plate center
ν	= Poisson's ratio

THE deformation of the side wall and its edge beams of an air plane cargo box during aircraft maneuvering may become excessive and intolerable. A rigorous analysis of the problem, which basically involves the determination of the deformation of plates supported by edge beams elastic in the plane of the plate, will be very difficult. An engineering approach for estimating the center deflections of a uniformly loaded square plate and the midsection deflection of the edge beam is presented. The side wall of a typical C-5A cargo box subjected to statically equivalent uniformly distributed inertia load during aircraft maneuvering is used as an example.

Received November 9, 1970. The work was a part of consulting work performed at the Lockheed-Georgia Company, Marietta, Ga.

* Professor, School of Engineering Science and Mechanics.

In this study, all materials are considered to be elastic, identical uniform edge beams are assumed to be perfectly rigid in the transverse direction and elastic in the plane of the plate, and only bending of edge beams in the plane of plate is considered. For uniformly loaded plate, the center deflection that represents the maximum deflection will be the main interest of this study.

The largest (upper bound) deflection w_L can be determined according to the bending theory of plate, i.e., when zero rigidity of edge beams in the plane of plate and no membrane effects are considered. The deflection of the plate may be generally represented as follows:

$$w_L = \alpha p(2a)^4/Df(x,y) \tag{1}$$

where a is the half plate width, p is the transverse loading, D is the plate rigidity, $f(x,y)$ represents the mode of deformation, and $f(0,0) = 1$. The coefficient α depends on the edge supporting conditions. According to Ref. 1 (see Tables 8 and 35),

$$\alpha = 0.00406 \text{ for simply supported plates} \tag{2a}$$

$$\alpha = 0.00126 \text{ for clamped plates} \tag{2b}$$

The maximum deflection is therefore

$$W_L = w_L(0,0) = \alpha p(2a)^4/D \tag{3}$$

The smallest (lower bound) deflection w_s can be determined by considering infinitely rigid edge beams and taking into account the membrane effects. The center deflection according to Ref. 1 (see pp. 422–424) may be obtained from the following equation:

$$p = [W_sD/\alpha(2a)^4] + W_s{}^3EH/0.516a^4 \tag{4}$$

where EH is the extensional rigidity. For a square plate with elastic edge beams as being considered in this study, the center deflection W_o will lie between W_L and W_s or

$$W_s < W_o < W_L \tag{5}$$

Square Plates with Elastic Edge Beams

Let U be the final deflection of the edge beam. The corresponding displacement W^* due to a portion p^* of the total load p, without consideration of the membrane effects is

$$w^* = \zeta W_of(x,y) = \alpha p^*(2a)^4/Df(x,y) \tag{6}$$

The function $f(x,y)$ satisfying the boundary conditions may be represented as follows:

$$f(x,y) = \cos\pi x/2a \cos\pi y/2a \tag{7a}$$

for a four sides simply supported plate, and

$$f(x,y) = \tfrac{1}{4}(1 + \cos\pi x/a)(1 + \cos\pi y/b) \tag{7b}$$

for a four sides clamped plate. The displacement U of an

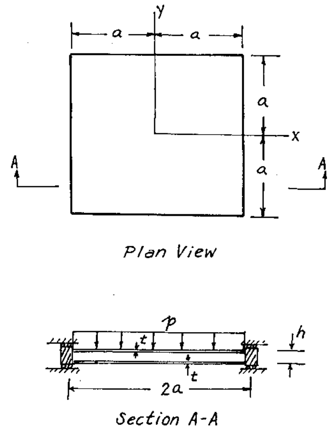

Fig. 1 Geometry and coordinates.